Name ______________ **Your Signature** ______________

Instructions:
1. For writing your answers use both sides of the paper in the answer booklet.

2. Additional sheets taken, if any, should be properly attached to the main answer booklet.

3. **Please write your name on every page of this booklet and every additional sheet taken.**
4. Maximum time is 3 hours

Score

Q.No.	**Alloted Score**	**Score**
1.	(15 points)	
2.	(15 points)	
3.	(20 points)	
4.	(20 points)	
5.	(15 points)	
6.	(15 points)	
Total	100	

Number of Extra sheets attached to the answer script: ______________

1. (15 points) Let $X_1, X_2, \ldots, X_n$ be i.i.d. samples from Poisson (λ). Find the maximum likelihood estimate for λ.

2. (15 points) Suppose we toss a fair coin n times and let $X_i = 1$ if head occurs at the i-th toss and 0 otherwise. Let

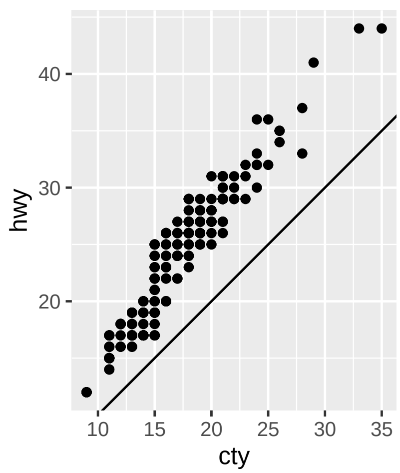
$$\hat{p}_n = \frac{|\{i : X_i = 1\}|}{n}.$$

Find $E[\hat{p}_n]$ and $Var(\hat{p}_n)$

3. Solve the following questions:

(a) (5 points) What does the plot below tell you about the relationship between city and highway mpg? Why is coord_fixed() important? What does geom_abline() do?

```
> library(tidyverse)
> ggplot(data = mpg, mapping = aes(x = cty, y = hwy)) +
+   geom_point() +
+   geom_abline() +
+   coord_fixed()
```



(b) (8 points) Let X be a Normal Random variable with mean μ and standard deviation σ. Compute

i. Compute $P(| X - 4 |< 3)$ when $\mu = 4$, $\sigma = 6$

ii. Compute $P(| X - 2 |< 12)$ when $\mu = 2$, $\sigma = 4$

by using the Normal tables provided.

(c) (7 points) Suppose the group learned-tossers perform 10 independent trials of tossing a biased coin with probability of heads p, 50 times. Find the likelihood (or probability) of obtaining a maximum of 30 heads in 10 independent trials of 50 tosses of biased coin with probability of heads p.

4. Solve the following questions.

   (a) (10 points) For the R-code below, describe the: null hypothesis, alternate hypothesis, the test statistic, the result of the test (with appropriate justification).

```
> x <- c(0.22, 0.8, -0.24, 0.18, 0.16, -0.67, 1.19, -0.07, -0.13, -0.03)
> y <- c(0.19, 3.00, 1.34, 1.35, 1.95, 1.63, 0.68, 0.51, 1.65, 0.92)
> wilcox.test(x, y)

        Wilcoxon rank sum exact test

data:  x and y
W = 8, p-value = 0.0007253
alternative hypothesis: true location shift is not equal to 0
```

(b) (10 points) The R-code below considers a sample from a Normal population:

$$175, 176, 173, 175, 174, 173, 173, 176, 173, 179$$

Using the outputs of appropriate computation below (with appropriate justification):
(i) describe a confidence interval for the mean of the distribution
(ii) decide on the outcome of the test $H_0 : \mu \geq 175$ vs $H_0 : \mu < 175$

```
> x = c(175, 176, 173, 175, 174, 173, 173, 176, 173, 179)
> t = sqrt(10)*(mean(x) -175)/(sd(x))
```

`> pt(t, 9)`	`> qt(0.975,9)`	
`[1] 0.3188211`	`[1] 2.262157`	`> sqrt(10)`
`> mean(x)`	`> sd(x)`	`[1] 3.162278`
`[1] 174.7`	`[1] 1.946507`	

5. (15 points) Suppose that U and V are independent Uniform$(0, 1)$, and interpret them as coordinates of a point in $\mathbb{R}^2$. We wish to estimate the probability that the point (U, V) is inside the unit circle.

   (a) Let $Z = \sqrt{U^2 + V^2}$. Find[1] the $P(Z < 1)$.

   (b) Explain the computation performed in the below `R-code` and (justify) compare the output with answer obtained in part (a).

```
> set.seed(2024)
> I = replicate(10, {
+   u <- runif(10000)
+ v <- runif(10000)
+ sum(u^2+v^2 < 1) / 10000
+ })
> mean(I)

[1] 0.78669
```

   (c) Write an `R`-code, modifying the above code, that will estimate π. Provide what the modified code would provide as the estimate.

[1] you may use the fact the integral of constant function 1 over a region in $\mathbb{R}^2$ is the area of the region.

6. (15 points) In an experiment in breeding plants, a geneticist has obtained 219 brown wrinkled seeds, 81 brown roundseeds, 69 white wrinkled seeds and 31 white round seeds. Theory predicts that these types of seeds should be obtained in the ratios $9 : 3 : 3 : 1$. Assuming that the null hypothesis is given by the theory, execute a test that can check if there is enough evidence to reject the null hypothesis at 5% level of significance.

Name (Please Print) ____________________

Statistical Tables for Reference if Needed.

Table 1: in (i, j)-th entry provides $\mathbb{P}(\chi_i^2 \leq j)$, with $\chi_i^2 \sim$ Chi-square with i degrees of freedom.

	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
2	0.181	0.221	0.259	0.295	0.330	0.362	0.393	0.423	0.451	0.478	0.503
3	0.060	0.081	0.104	0.127	0.151	0.175	0.199	0.223	0.247	0.271	0.294
4	0.018	0.026	0.037	0.049	0.062	0.075	0.090	0.106	0.122	0.139	0.156
5	0.005	0.008	0.012	0.017	0.023	0.030	0.037	0.046	0.055	0.065	0.076
6	0.001	0.002	0.004	0.006	0.008	0.011	0.014	0.018	0.023	0.028	0.034
7	0.000	0.001	0.001	0.002	0.003	0.004	0.005	0.007	0.009	0.012	0.014
8	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.002	0.003	0.004	0.006
9	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.002	0.002

Table 2: in (i, j)-th entry provides $\mathbb{P}(\chi_i^2 \leq j)$, with $\chi_i^2 \sim$ Chi-square with i degrees of freedom.

	2.50	2.52	2.54	2.56	2.58	2.60	2.62	2.64	2.66	2.68	2.70
1	0.886	0.888	0.889	0.890	0.892	0.893	0.894	0.896	0.897	0.898	0.900
2	0.713	0.716	0.719	0.722	0.725	0.727	0.730	0.733	0.736	0.738	0.741
3	0.525	0.528	0.532	0.535	0.539	0.543	0.546	0.549	0.553	0.556	0.560
4	0.355	0.359	0.363	0.366	0.370	0.373	0.377	0.380	0.384	0.387	0.391
5	0.224	0.227	0.230	0.233	0.236	0.239	0.242	0.245	0.248	0.251	0.254
6	0.132	0.134	0.136	0.138	0.141	0.143	0.145	0.148	0.150	0.152	0.155
7	0.073	0.074	0.076	0.077	0.079	0.081	0.082	0.084	0.085	0.087	0.089
8	0.038	0.039	0.040	0.041	0.042	0.043	0.044	0.045	0.046	0.047	0.048
9	0.019	0.020	0.020	0.021	0.021	0.022	0.023	0.023	0.024	0.024	0.025
10	0.009	0.009	0.010	0.010	0.010	0.011	0.011	0.011	0.012	0.012	0.012

Table 3: in (i, j)-th entry provides $\mathbb{P}(\chi_i^2 \leq j)$, with $\chi_i^2 \sim$ Chi-square with i degrees of freedom.

	2.50	2.52	2.54	2.56	2.58	2.60	2.62	2.64	2.66	2.68	2.70
2	0.713	0.716	0.719	0.722	0.725	0.727	0.730	0.733	0.736	0.738	0.741
3	0.525	0.528	0.532	0.535	0.539	0.543	0.546	0.549	0.553	0.556	0.560
4	0.355	0.359	0.363	0.366	0.370	0.373	0.377	0.380	0.384	0.387	0.391
5	0.224	0.227	0.230	0.233	0.236	0.239	0.242	0.245	0.248	0.251	0.254
6	0.132	0.134	0.136	0.138	0.141	0.143	0.145	0.148	0.150	0.152	0.155
7	0.073	0.074	0.076	0.077	0.079	0.081	0.082	0.084	0.085	0.087	0.089
8	0.038	0.039	0.040	0.041	0.042	0.043	0.044	0.045	0.046	0.047	0.048
9	0.019	0.020	0.020	0.021	0.021	0.022	0.023	0.023	0.024	0.024	0.025

Normal Tables

$\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp(-\frac{y^2}{2})$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	0.5398
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	0.5793
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	0.6179
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	0.6554
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	0.6915
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	0.7257
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	0.7580
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	0.7881
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	0.8159
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	0.8413
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	0.8643
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	0.8849
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	0.9032
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	0.9192
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	0.9332
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	0.9452
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	0.9554
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	0.9641
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	0.9713
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	0.9772
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0.9821
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0.9861
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0.9893
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0.9918
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0.9938
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0.9953
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0.9965
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0.9987
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998